Sub-Nyquist Wideband Acquisition and Spectrum Sensing of Multiband Signals

2015 Wireless Innovation Forum
Spectrum Sharing Workshop
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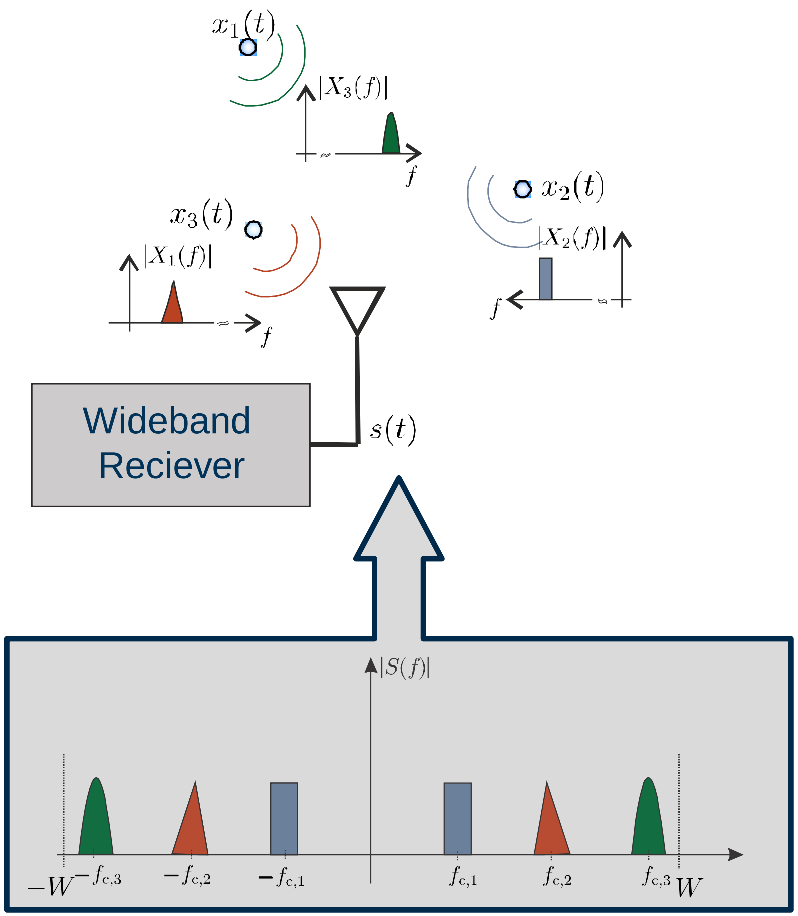
Anastasia Lavrenko, Florian Römer, Giovanni Del Galdo and Reiner Thomä

Introduction



Wideband **M**ultiband **S**ignal

- **few narrowband** communication **channels** distributed over **wide frequency band** of interest W.

Wideband **S**pectrum **S**ensing

- find out which **parts** of **spectrum** are **occupied** and which are not.

Blind **W**ideband **S**pectrum **S**ensing

- **unknown** carrier frequencies

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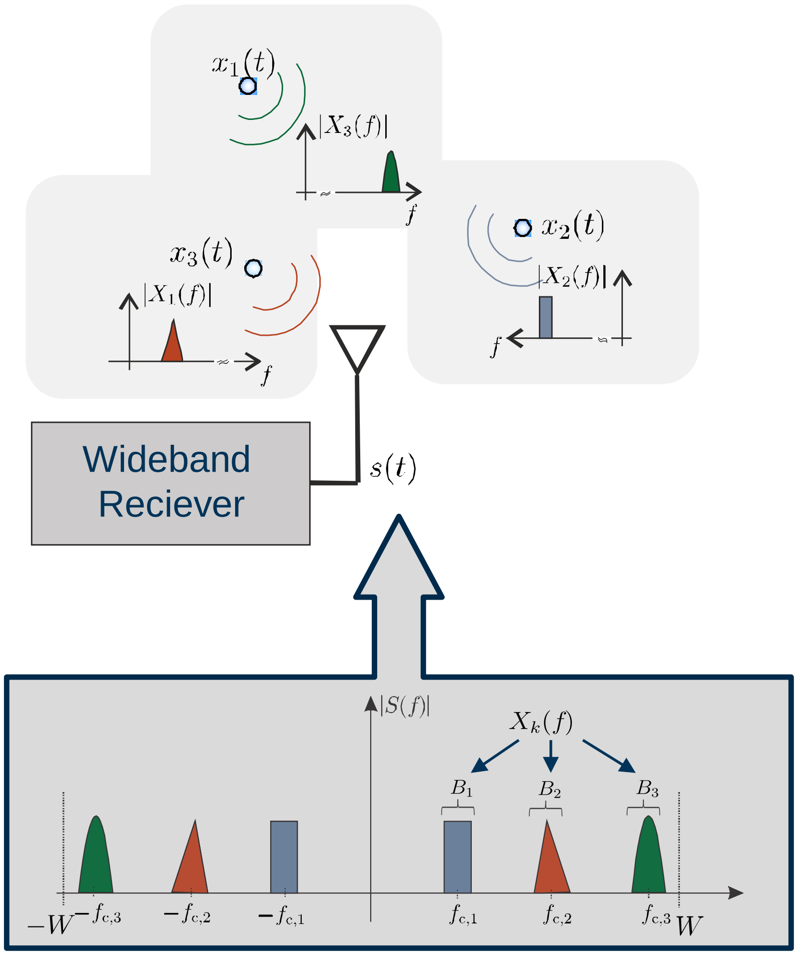

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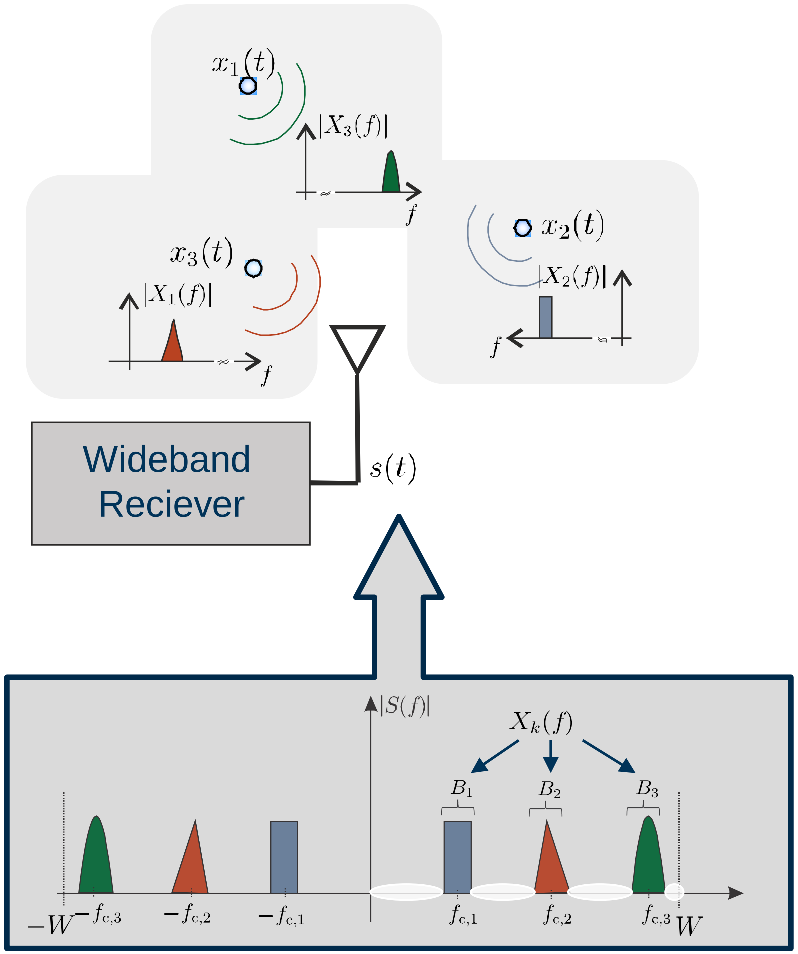

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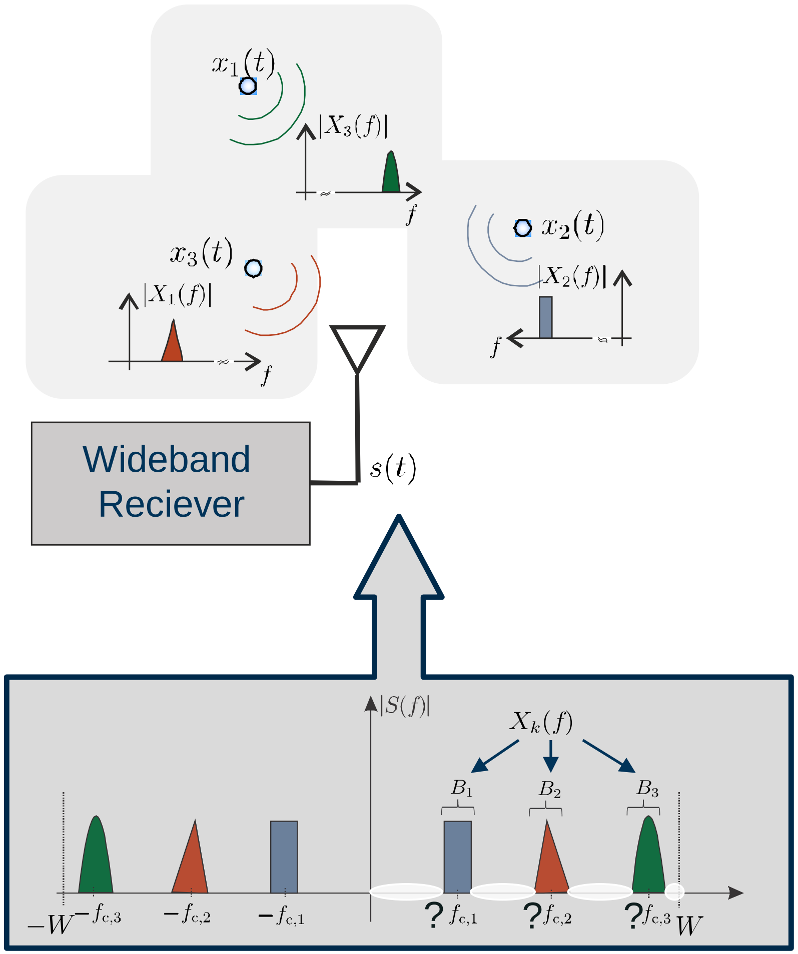

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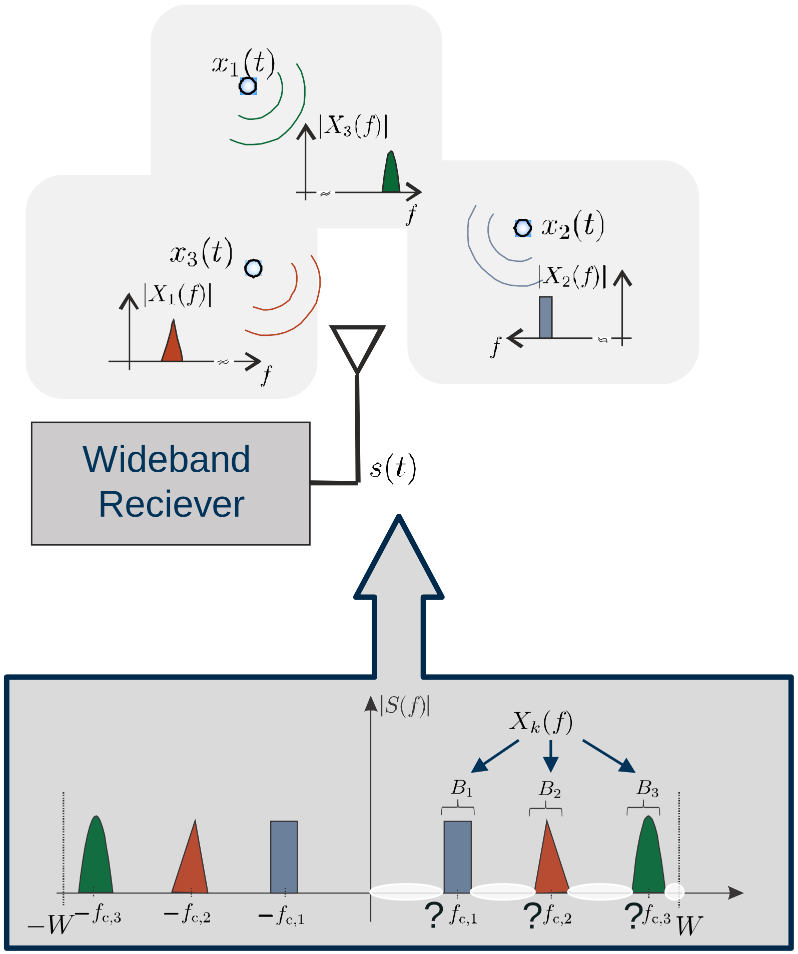

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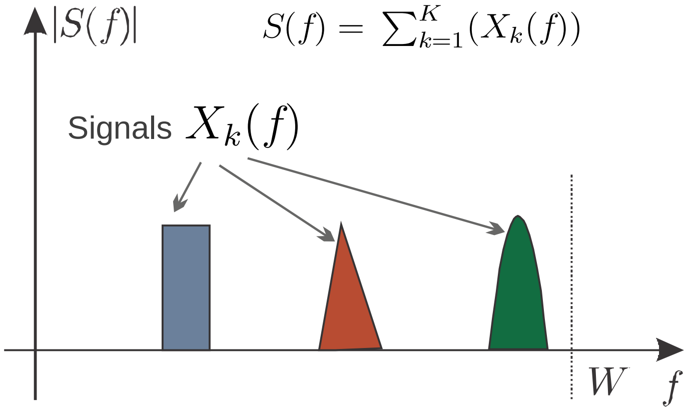
- Q1: can one **receive** such a wideband signal sampling it lower than required by Nyquist theorem?
- Q2: how can one **detect** spectral occupancy from the received time samples?
- Q3: how **accurate** this detection is?

Outline

- Sub-Nyquist Signal Acquisition
  - main principle
  - examples of architectures
- Compressive Wideband Spectrum Sensing
  - Nyquist-sampled signal reconstruction
  - PSD reconstruction
  - direct course energy estimation
- Numerical examples

Sub-Nyquist Signal Acquisition: Paradigm



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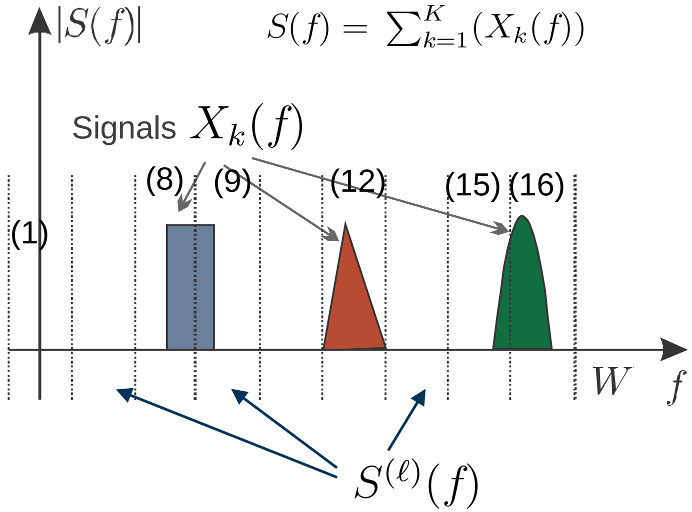

- Vitrually **split** the **Nyquist range** into L consecutive **non-overlapping blocks** (spectral cells) of width W/L.
- Treat **wideband signal** $S(f)$ as a finite **union** of continuous **bandpass** (narrowband) **sub-spaces**

$$S(f) = [S^{(1)}(f), S^{(2)}(f), \ldots, S^{(L)}(f)]$$

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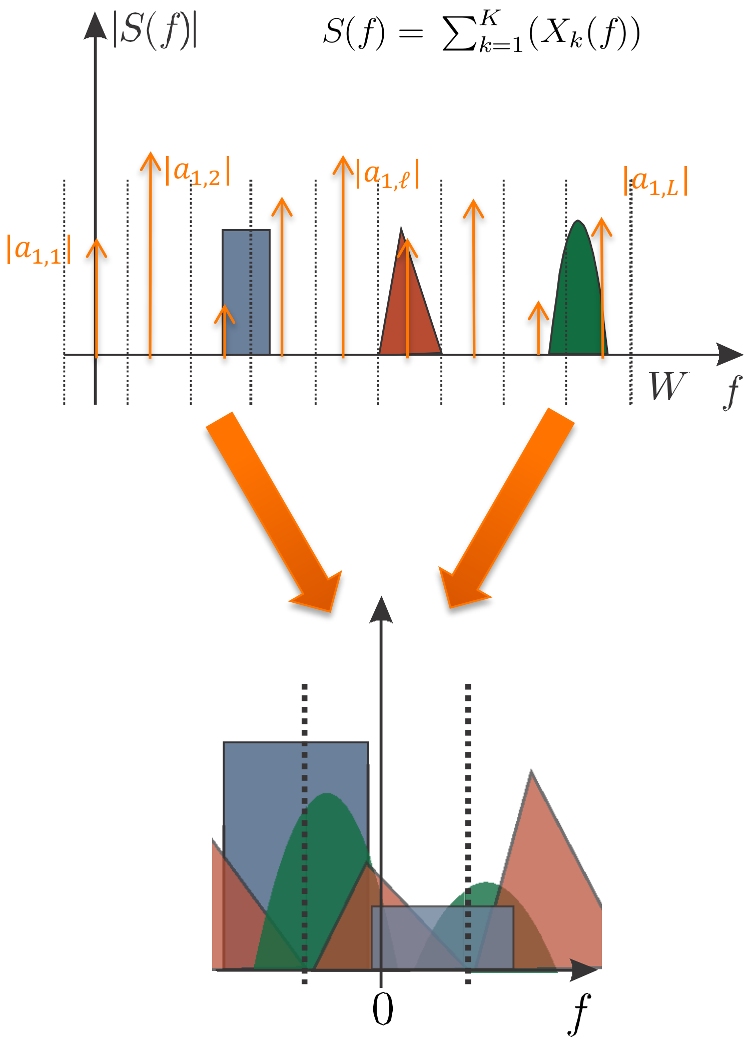

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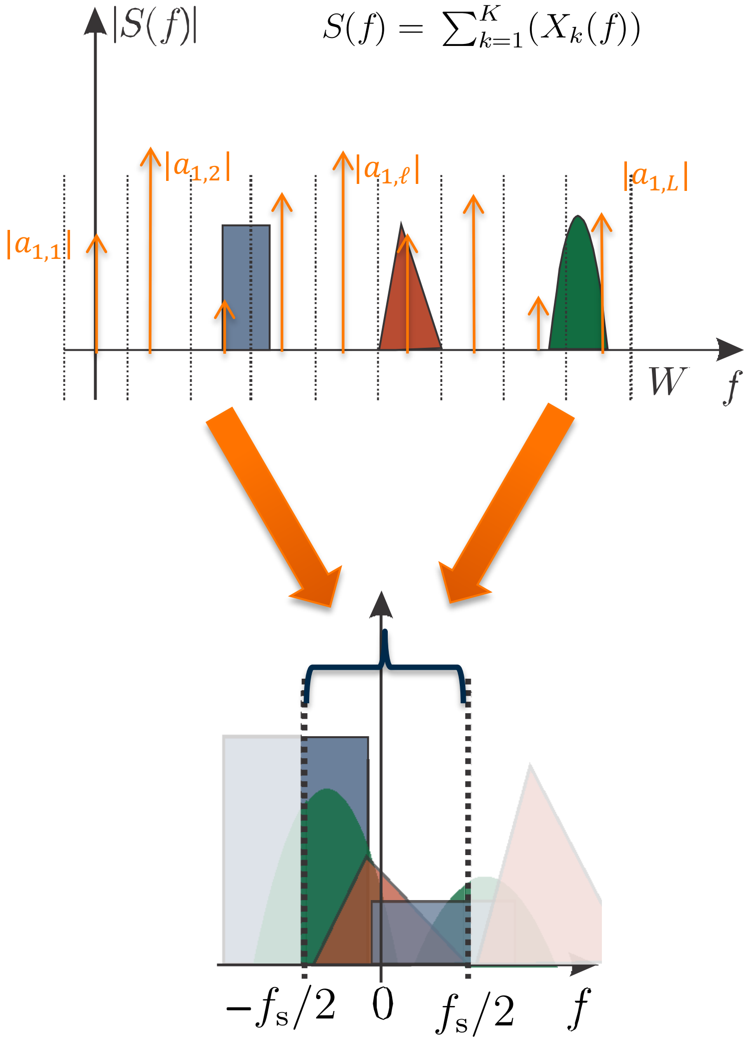

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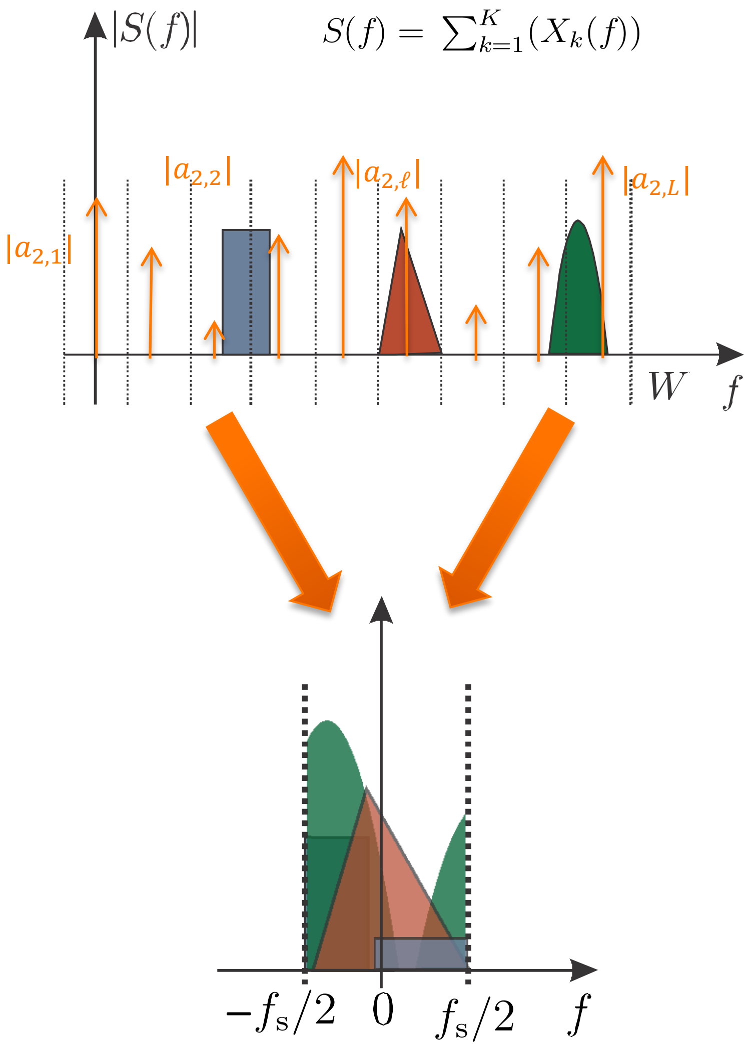

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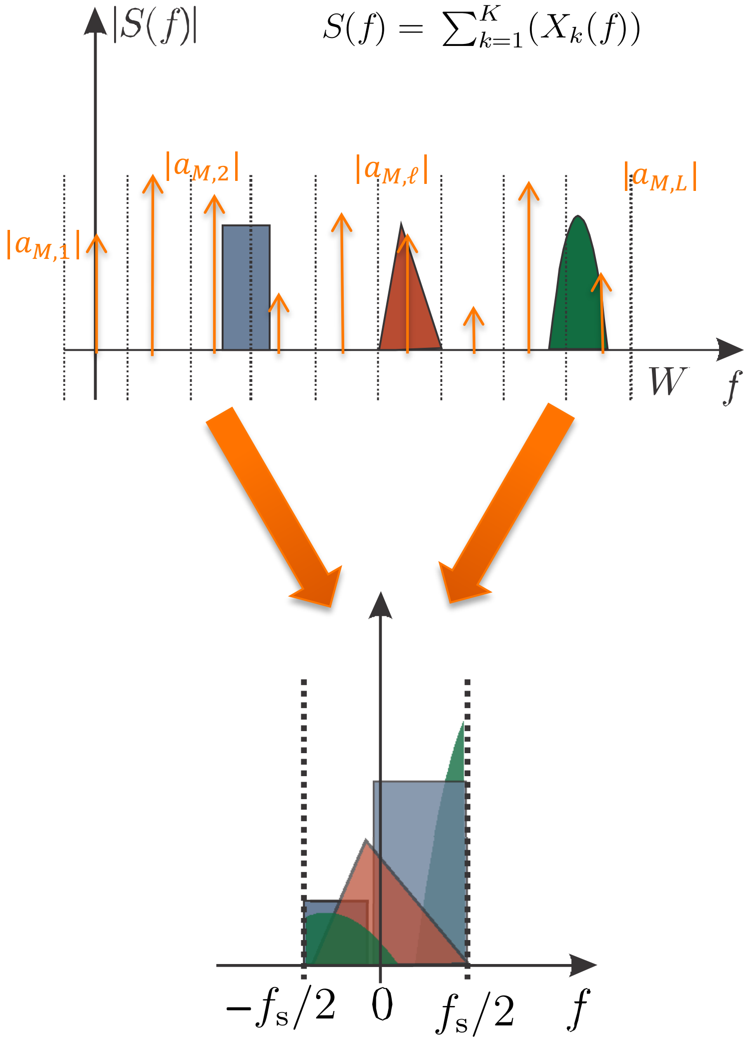

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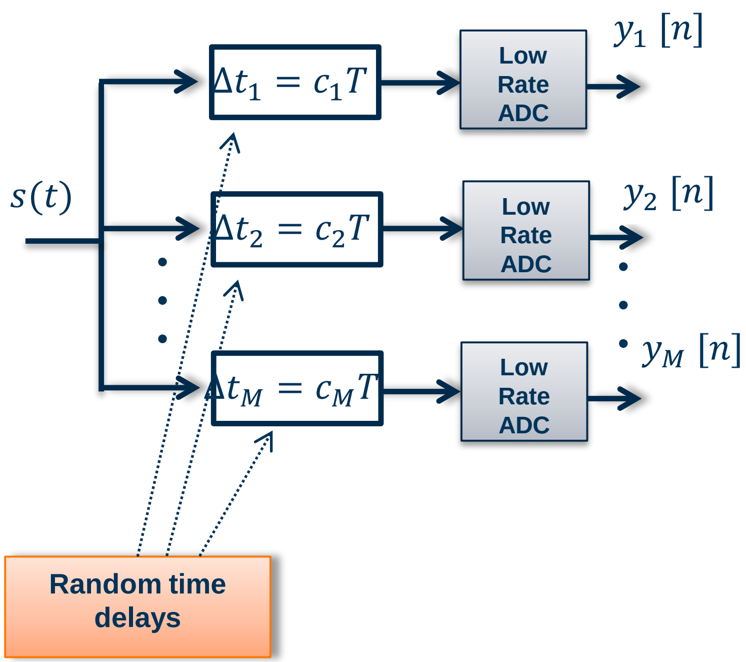
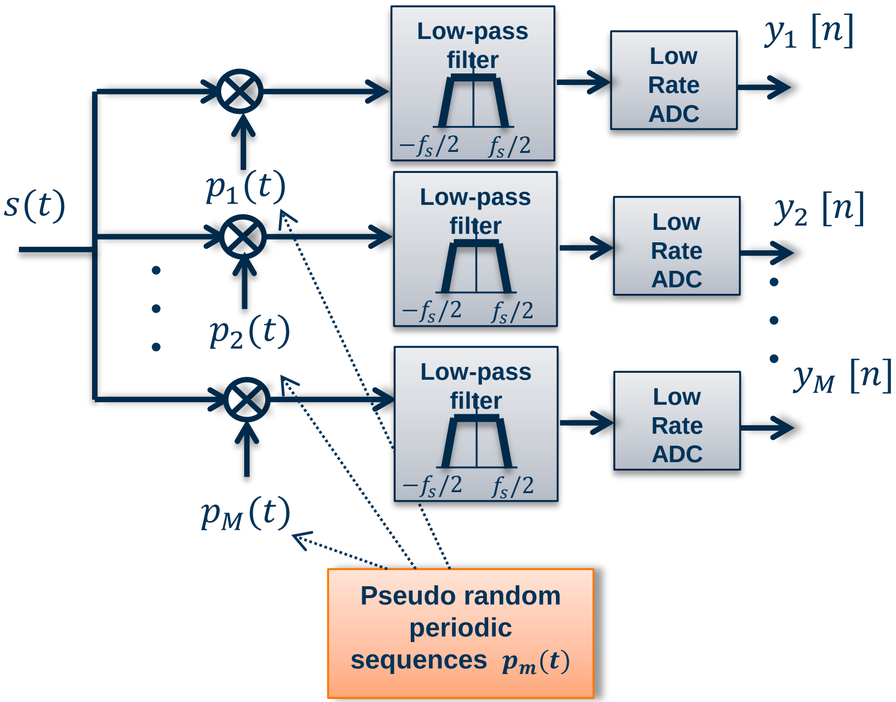
Sub-Nyquist Signal Acquisition: Sampling Architectures

Periodic Non-Uniform Sampling‘



Modulated Wideband Converter*



‘ Bresler, Y. (2008, January). Spectrum-blind sampling and compressive sensing for continuous-index signals. In *Information Theory and Applications Workshop, 2008* (pp. 547-554). IEEE.

* Mishali, M., & Eldar, Y. C. (2010). From theory to practice: Sub-Nyquist sampling of sparse wideband analog signals. Selected Topics in Signal Processing, IEEE Journal of, 4(2), 375-391.

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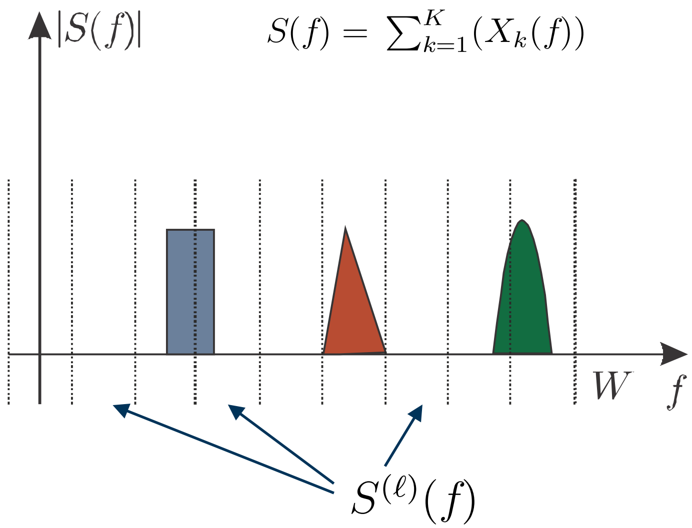
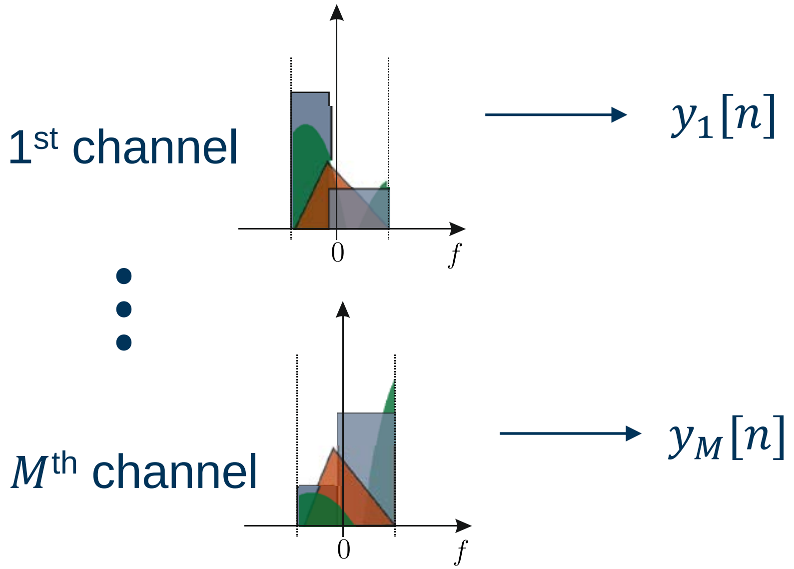
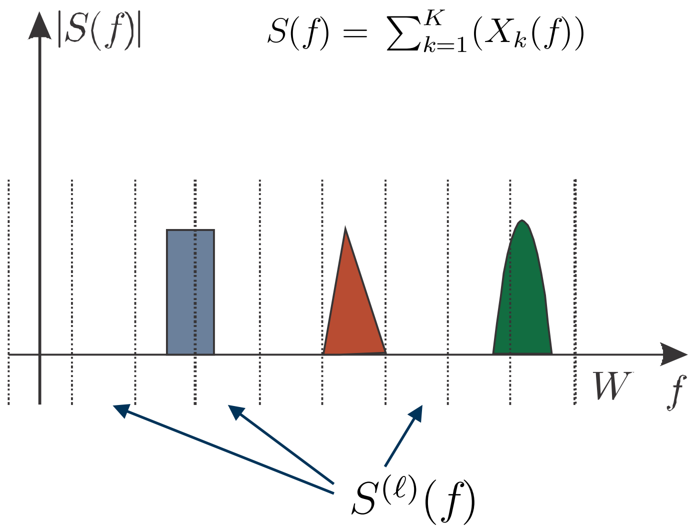
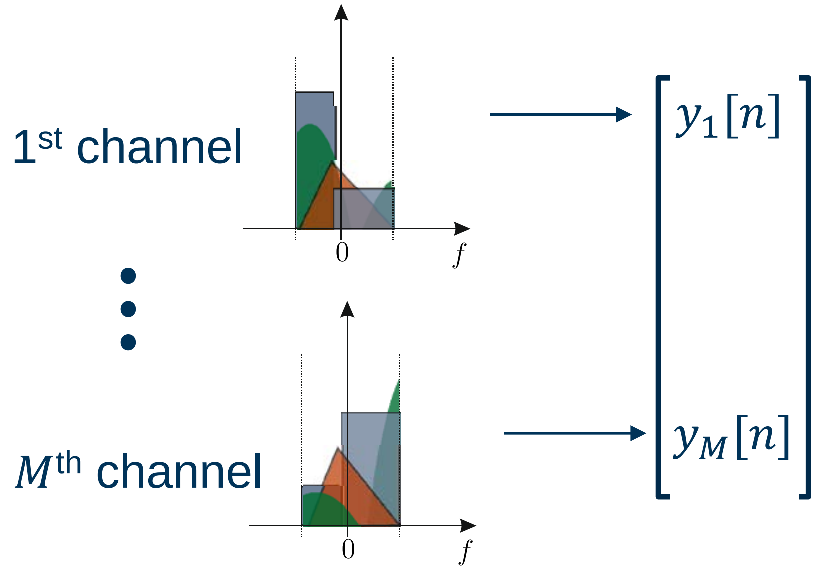
Signal recovery





Signal Recovery





Output ‘compressed’ signal

$$\boldsymbol{y}[n] = \boldsymbol{A}\cdot\boldsymbol{z}[n]$$

IDTFT ⇕ DTFT

$$\dot{\boldsymbol{y}}(f) = \boldsymbol{A}\cdot\dot{\boldsymbol{z}}(f)$$

$$\boldsymbol{A} = \begin{bmatrix} a_{1,1} & \cdots & a_{1,L} \\ \vdots & \ddots & \vdots \\ a_{M,1} & \cdots & a_{M,L} \end{bmatrix}$$

$$\dot{\boldsymbol{z}}(f) = \begin{bmatrix} S^{(1)}(f - f_s) \\ \vdots \\ S^{(L)}(f - Lf_s) \end{bmatrix}, f \in [-\frac{f_s}{2}, \frac{f_s}{2}]$$

Signal Recovery

- Consider and observation covariance matrix $\boldsymbol{R}$

$$\boldsymbol{R} = \sum_{n=-\infty}^{\infty} \boldsymbol{y}[n] \cdot \boldsymbol{y}^{\mathrm{H}}[n] = \int_{f \in \mathcal{F}_\mathrm{s}} \dot{\boldsymbol{y}}(f) \cdot \dot{\boldsymbol{y}}^{\mathrm{H}}(f), \quad \mathcal{F}_\mathrm{s} = [-f_\mathrm{s}/2, f_\mathrm{s}/2]$$

- Taking into account $\boxed{\dot{\boldsymbol{y}}(f) = \boldsymbol{A} \cdot \dot{\boldsymbol{z}}(f)}$

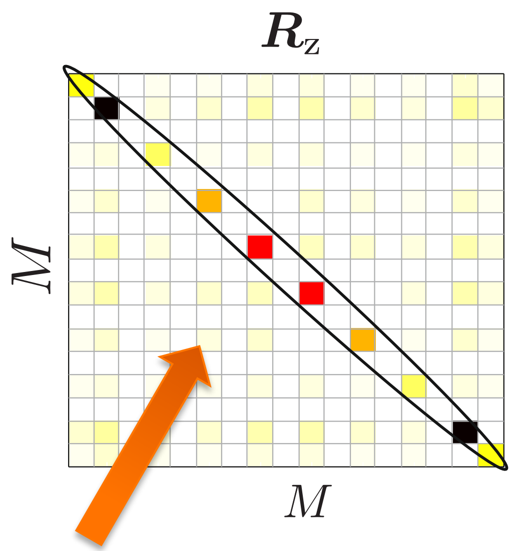
$$\boldsymbol{R} = \int_{f \in \mathcal{F}_\mathrm{s}} \boldsymbol{A} \cdot \dot{\boldsymbol{z}}(f) \cdot \dot{\boldsymbol{z}}^{\mathrm{H}}(f) \cdot \boldsymbol{A}^{\mathrm{H}} \mathrm{d}f = \boldsymbol{A} \cdot \boldsymbol{R}_\mathrm{z} \cdot \boldsymbol{A}^{\mathrm{H}}$$

where

- **diagonal** elements of $\boldsymbol{R}_\mathrm{z}$ represent the **energy** of the corresponding **spectral cell**

$$(R_\mathrm{z})_{jj} = \int_{f \in \mathcal{F}_\mathrm{s}} |\dot{z}_j(f)|^2 \mathrm{d}f = \int_{f \in \mathcal{F}^j} |S^{(j)}(f)|^2 \mathrm{d}f$$

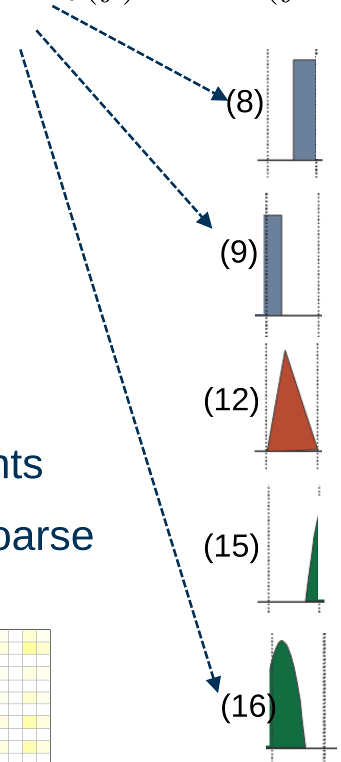
- **non-diagonal** – cross-correlation between spectral sells ⇒ are **equal to 0** assuming that the individual signals are independent from each other



$\tilde{K} \leq 2K$ non-zero elements

Signal Recovery

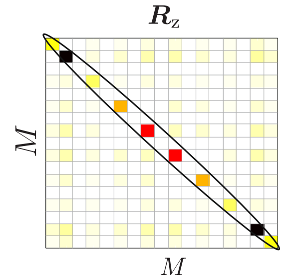
- Denote $\mathcal{S}$ the support of $\dot{\mathbf{z}}(f)$ – indices corresponding to non-zero $z_\ell(f) = S^{(\ell)}(f - \ell f)$



- Apply **sparse recovery** approaches from CS to estimated $\mathcal{S}$!

- After the positions of the occupied spectral blocks are estimated
  - recover the time samples at the Nyquist rate and perform conventional spectral analysis
  - estimate the PSD directly from the sub-Nyquist measurements
  - directly interpret the results of the support recovery as the coarse spectral occupancy detection

Non-zero elements of the covariance matrix are energies of the occupied spectral blocks



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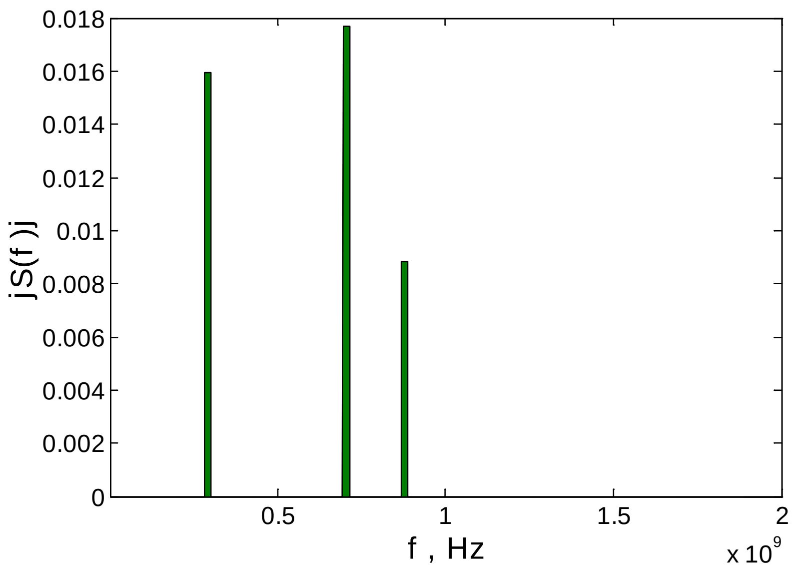
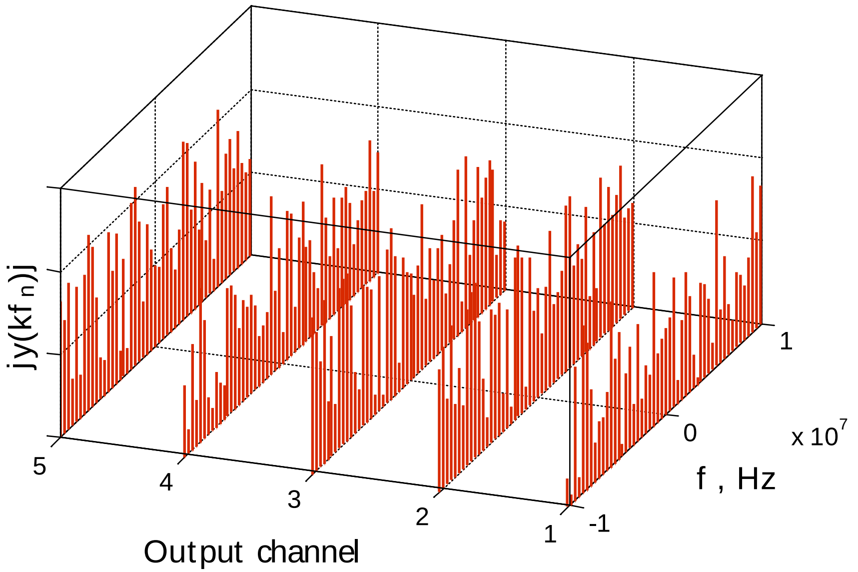
Numerical examples. Simulation setup

Signal and sampling parameters

- Test multiband signals were generated in the frequency domain
- The sensing matrix $\boldsymbol{A}$ was chosen randomly with entries drawn independently from a complex zero-mean normal distribution

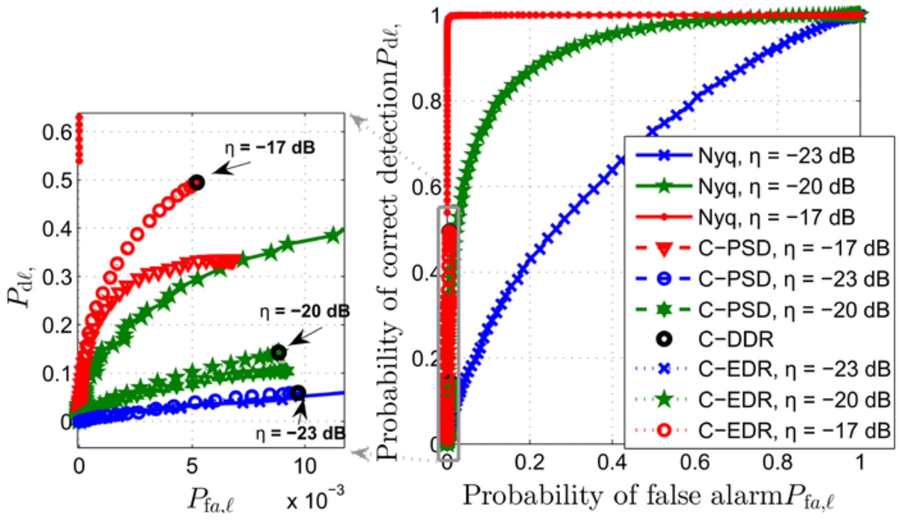
Signal Parameters	Sampling Parameters
$K \in [1, 3]$	$f_{\mathrm{NYQ}}/B = 105$
$B = 19$ MHz	$L = 97 \leq f_{\mathrm{NYQ}}/B$
$f_{\mathrm{NYQ}} = 2$ GHz	$f_{\mathrm{s}} \approx 20.6$ MHz
$N = 50$	$M = 30$



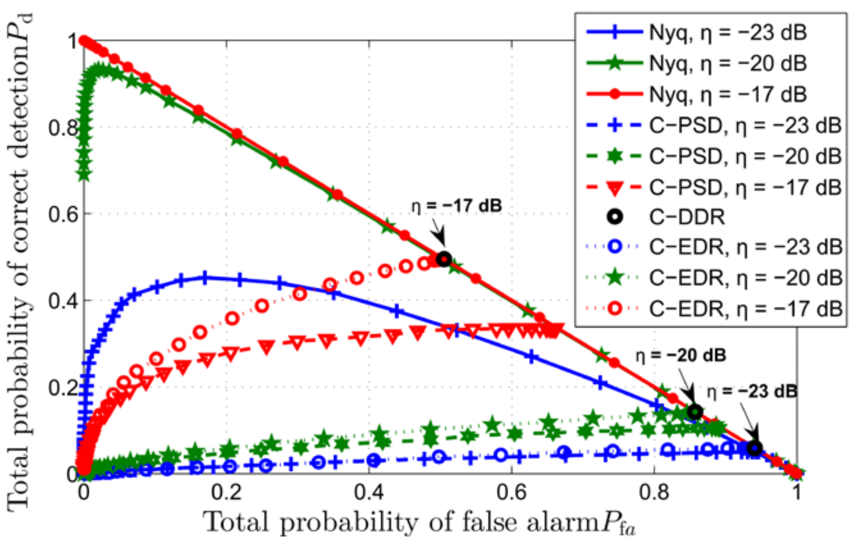


Numerical results. Single occupied spectral cell

Probability of correct detection per block vs. probability of false alarm per block

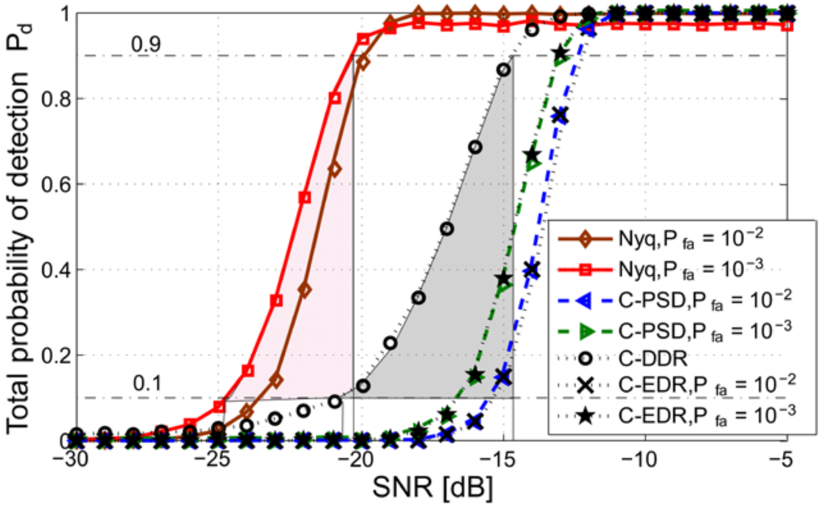


Total probability of correct detection vs. total probability false alarm

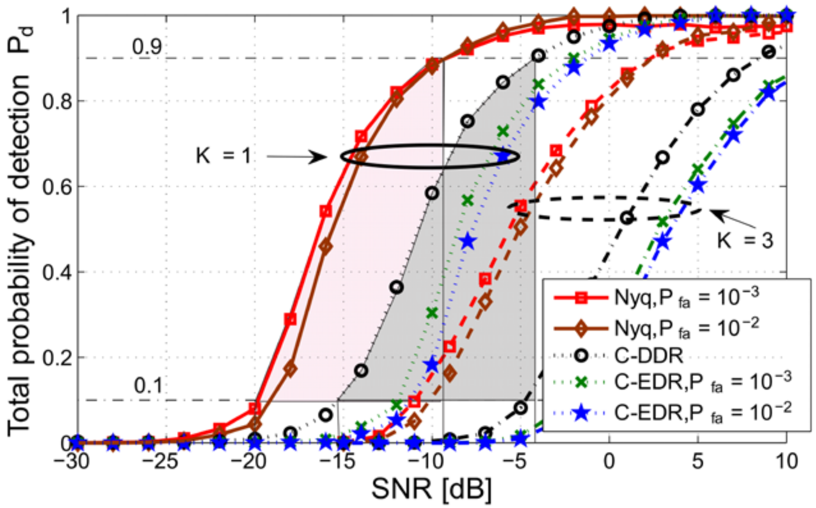


Numerical results. Single and multiple occupied spectral cells

Total probability of correct detection vs. SNR for a single occupied spectral cell



Total probability of correct detection vs. SNR with no restrictions on the signal sub-band positions



Conclusions

- We’ve shown
  - how wideband spectrum sensing can be performed at the **sub-Nyquist** sampling rates.
  - that it is **possible** to infer the **coarse spectral occupancy** directly from the results of the **support** recovery, i.e., without the need to recover full signal, its spectrum or PSD.
- Sub-Nyquist wideband spectrum sensing provides a viable approach to **‘fast and dirty’** estimation of the spectral occupancy over wide frequency ranges in highly underutilized environments.

Thank you for your attention!

Anastasia Lavrenko
Electronic Measurement lab
TU Ilmenau, Germany
anastasia.lavrenko@tu-ilmenau.de

Contact Information

- M.Sc. Anastasia Lavrenko

  PhD Student in the *International Graduate School on Mobile Communications*
  Electronic Measurement Research Lab
  Ilmenau University of Technology

  office: +49 3677 69 1397
  e-mail: anastasia.lavrenko@tu-ilmenau.de

Mailing address:
Ilmenau University of Technology,
Helmholzplatz 2
98693 Ilmenau, Germany

- Dr.-Ing. Florian Römer

  Digital Broadcasting Research Laboratory,
  Ilmenau University of Technology

  office: +49 3677 69 4286
  e-mail: florian.roemer@tu-ilmenau.de

- Prof. Giovanni Del Galdo

  Head of DVT Research Group, composed of
  Digital Broadcasting Research Laboratory,
  Ilmenau University of Technology
  Wireless Distribution Systems / Digital Broadcasting Group,
  Fraunhofer Institute for Integrated Circuits IIS

  office: +49 3677 69-4280
  e-mail: giovanni.delgaldo@tu-ilmenau.de
  giovanni.delgaldo@iis.fraunhofer.de

- Prof. Reiner Thomä

  Head of Electronic Measurement Research Laboratory
  Ilmenau University of Technology

  office: + 49 3677 69-2622
  e-mail: reiner.thomae@tu-ilmenau.de

Spectral Recovery

- Denote $\mathcal{S}$ the support of $\dot{\boldsymbol{z}}(f)$ – indices corresponding to non-zero $z_\ell(f) = S^{(\ell)}(f - \ell f)$

- Construct a frame $\boldsymbol{V}$ such that

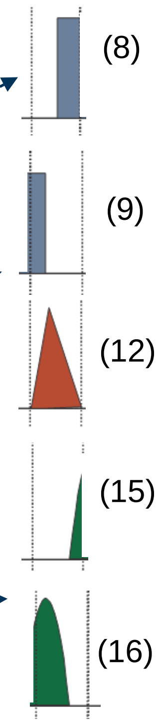
$$\boldsymbol{V} \cdot \boldsymbol{V}^{\mathrm{H}} = \hat{\boldsymbol{R}} = \sum_{n=1}^{N:N>2K} \boldsymbol{y}[n] \cdot \boldsymbol{y}[n]^{\mathrm{T}}$$

- Solving $\boldsymbol{V} = \boldsymbol{A} \cdot \boldsymbol{U}$ for sparsest $\boldsymbol{U}$ yields a matrix with support $\mathcal{S}$!

block sparse recovery

- Once support $\mathcal{S}$ is found,

$$\begin{cases} \boldsymbol{z}_{\mathcal{S}}[n] = \boldsymbol{A}_{\mathcal{S}}^{\dagger} \cdot \boldsymbol{y}[n] \\ z_i[n] = 0, \ i \notin \mathcal{S} \end{cases}$$



- Generate Nyquist rate sampled sequences and calculate DFT
- Apply DFT to $\boldsymbol{z}[n]$ and reshape it into one vector

PSD Recovery

- Denote $P_s(f) = \int_{-\infty}^{\infty} \mathsf{E}\{s(t)\cdot s(t-\tau)\}e^{-\jmath 2\pi f\tau}\mathrm{d}\tau$
- If $s(t)$ is wide-sense stationary, then $\boldsymbol{R}_z(f) = \mathsf{E}\{\boldsymbol{z}(f)\cdot\boldsymbol{z}^{\mathrm{H}}(f)\}$ is diagonal and

$$\boxed{R_{z_{(i,i)}}(f) = P_s(f + (i-1)f_s)}$$

- Now $\boldsymbol{R}_y(f) = \mathsf{E}\{\boldsymbol{y}(f)\cdot\boldsymbol{y}(f)^{\mathrm{H}}\} = \boldsymbol{A}\cdot\boldsymbol{R}_z(f)\cdot\boldsymbol{A}^{\mathrm{H}}$

and

$$\mathrm{vec}(\boldsymbol{R}_y(f)) = (\boldsymbol{A}'\otimes\boldsymbol{A})\mathrm{vec}(\boldsymbol{R}_z(f)) = (\boldsymbol{A}'\odot\boldsymbol{A})\mathrm{diag}(\boldsymbol{R}_z(f))$$

Kronecker & Khatri-Rao products

- Construct a frame
- Recover support $\mathcal{S}$ by solving a finite dimensional sparse recovery problem
- Invert $\boldsymbol{A}'_{\mathcal{S}}\odot\boldsymbol{A}_{\mathcal{S}}$ and reconstruct he power spectral densities $R_{z_{(i,i)}}$

Energy Recovery

Recap:

$$\boldsymbol{R} = \int_{f \in \mathcal{F}_\mathrm{s}} \boldsymbol{A} \cdot \dot{\boldsymbol{z}}(f) \cdot \dot{\boldsymbol{z}}^{\mathrm{H}}(f) \cdot \boldsymbol{A}^{\mathrm{H}} \mathrm{d}f = \boldsymbol{A} \cdot \boldsymbol{R}_\mathrm{z} \cdot \boldsymbol{A}^{\mathrm{H}}$$

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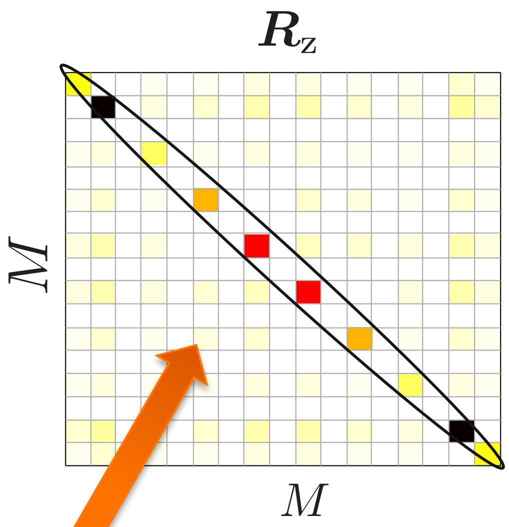


$\tilde{K} \leq 2K$ non-zero elements

- Let's vectorize the covariance matrix $\boldsymbol{R}$

$$\boldsymbol{r} = \mathrm{vec}(\boldsymbol{R}) = (\boldsymbol{A}' \otimes \boldsymbol{A}) \cdot \boldsymbol{r}_\mathrm{z} = (\boldsymbol{A}' \odot \boldsymbol{A}) \cdot \mathrm{diag}(\boldsymbol{R}_\mathrm{z})$$

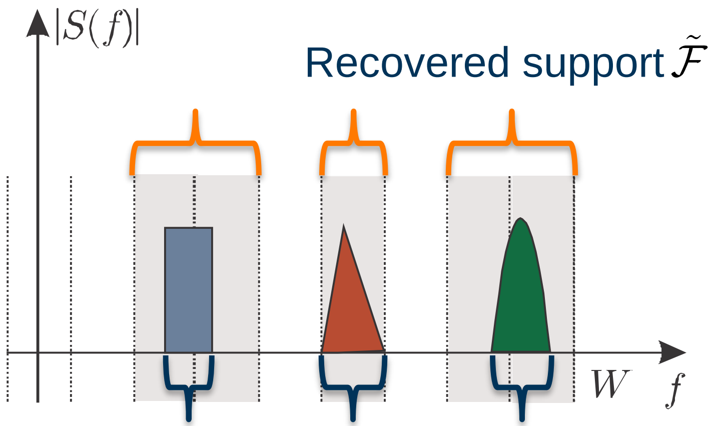
Kronecker & Khatri-Rao products

- Once the unknown support $\mathcal{S}$ is recovered, the energy $\boldsymbol{\varepsilon}_\mathcal{S}$ of the occupied cells is given by

$$\begin{cases} \boldsymbol{\varepsilon}_\mathcal{S} = \boldsymbol{\Psi}_\mathcal{S}^\dagger \cdot \boldsymbol{r} = (\boldsymbol{A}'_\mathcal{S} \odot \boldsymbol{A}_\mathcal{S})^\dagger \cdot \boldsymbol{r} \\ \varepsilon_i = 0, \ i \notin \mathcal{S} \end{cases}$$

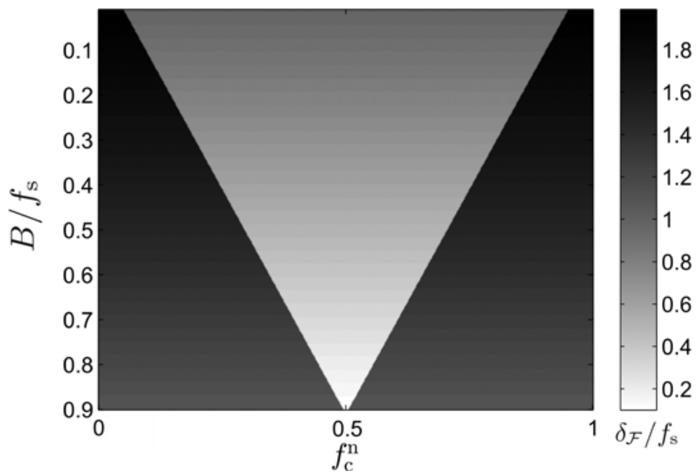
The results of the support recovery – coarse energy detection at the resolution of spectral cells

Wideband Spectrum Sensing



- In the blind case direct compressive ED provides a **coarse estimate** of the spectral occupancy – at the resolution of f_s

Normalized spectral support difference



- The **difference** between true and recovered spectral support **depends** on the **signal** parameters
- For an **unknown** uniformly distributed random signal spectral support each **signal sub-band** is split between **two** neighbouring **spectral cells** with a high probability

Numerical examples. Simulation setup

Performance metrics

- 'Per block' probabilities of false alarm and missed detection

$$P_{\mathrm{fa}\ell,} = Pr(b_\ell = 1|_{\int_{\mathcal{F}^\ell} |S^\ell(f)|^2 \mathrm{d}f = 0})$$

$$P_{\mathrm{md}\ell,} = Pr(b_\ell = 0|_{\int_{\mathcal{F}^\ell} |S^\ell(f)|^2 \mathrm{d}f > 0})$$

- Total probabilities of correct detection, false alarm and missed detection

$$P_d = Pr(b_\ell = 1 \forall \ell \in \mathcal{S},\, b_k = 0 \forall k \in \mathcal{S}')$$

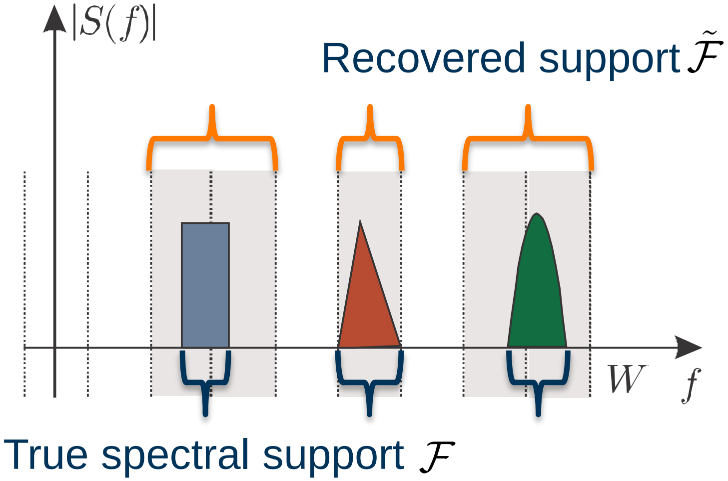
$$P_{\mathrm{fa}} = Pr(\exists k \in \mathcal{S}' \colon b_k = 1)$$

$$P_{\mathrm{md}} = Pr(\exists k \in \mathcal{S} \colon b_k = 0)$$

Evaluated multichannel ED approaches

- Power spectral density (PSD) based:
  - PSD recovered from $\boldsymbol{R}$ **– C-PSD**
  - PSD estimated from a Nyquist signal equivalent (no compression) **– Nyq**
- Compressive direct energy detection
  - direct decision rule – **C-DDR**
  - energy based decision rule – **C-EDR**
    1. first estimate the energy within the spectral cell
    2. apply classical ED

Wideband Spectrum Sensing



- In the blind case direct compressive ED provides a **coarse estimate** of the spectral occupancy – at the resolution of f_s

$$\boxed{0 \leq \delta_f = \lambda(\tilde{\mathcal{F}}) - \lambda(\mathcal{F}) = \tilde{K} f_\mathrm{s} - \sum_i B_i \leq 2K f_\mathrm{s}}$$